On this worksheet you will use substitution, as well as the other integration rules, to evaluate the the given definite and indefinite integrals.

Steps for integration by Substitution

1. Determine u: think parentheses and denominators
2. Find $\frac{du}{dx}$
3. Rearrange $\frac{du}{dx}$ until you can make a substitution
4. Make the substitution to obtain an integral in u
5. Integrate with respect to u
6. Substitute u back to be left with an expression in terms of x

Steps for finding the Definite Integral

1. Using substitution or otherwise, find an antiderivative $F(x)$
2. Using the given limits of integration, find $F(b) - F(a)$. Remember: b is the upper limit and a is the lower limit.

   Be careful to evaluate $-F(a)$ correctly (distribute the negative accordingly)

   Your answer should be a *number*

   If you make a substitution, remember to substitute back before plugging in your limits of integration

---

Example 1:

Find $\int 4x\left(x^2+1\right))^5\,dx$.

Observe that if $u = x^2+1$ then $\dfrac{du}{dx} = 2x$ and so

$$du = 2x\,dx \Longrightarrow 2\,du = 4x\,dx$$

$$\Longrightarrow \int 4x\left(x^2+1\right)^5\,dx = \int 2u^5\,du = \frac{1}{3}u^6 + C = \frac{1}{3}(x^2+1)^6 + C.$$

So $\int 4x\left(x^2+1\right)^5\,dx = \frac{1}{3}(x^2+1)^6 + C$.

Example 2:

Find $\int_1^5 9x^2+10x+3\,dx$.

$$\int_1^5 9x^2+10x+3\,dx = 3x^3+5x^2+3x\Big|_1^5 = 3(5)^3+5(5)^2+3(5) - \left[3(1)^3+5(1)^2+3(1)\right] = 515 - 11 = 504$$

So $\int_1^5 9x^2+10x+3\,dx = 504$.

Example 3:

Find $\int_0^2 \dfrac{2x}{\sqrt{x^2+4}}\,dx$.

Observe that if $u = x^2+4$ then $\dfrac{du}{dx} = 2x$ and so

$$du = 2x\,dx$$

$$\Longrightarrow \int_0^2 \frac{2x}{\sqrt{x^2+4}}\,dx = \int_{x=0}^{x=2} u^{-1/2}\,du = 2u^{1/2}\Big|_{x=0}^{x=2} = 2\left(x^2+4\right)^{1/2}\Big|_0^2 = 2\left(2^2+4\right)^{1/2} - 2\left(0^2+4\right)^{1/2} = 2\sqrt{8} - 4 = 4\left(\sqrt{2}-1\right).$$

So $\int_0^2 \dfrac{2x}{\sqrt{x^2+4}}\,dx = 4\left(\sqrt{2}-1\right)$.

---

1. $\int_1^2 \frac{1}{r^2}\,dr$
2. $\int \frac{3}{3x+5}\,dx$
3. $\int (4x+1)^8\,dx$
4. $\int_0^6 6y\,dy$
5. $\int 4x\left(2x^2+1\right)^5\,dx$
6. $\int_9^3 t^3\,dt$
7. $\int_0^1 \frac{3s^2+2}{2s^3+4s+3}\,ds$
8. $\int \frac{3x^2}{x^3+8}\,dx$
9. $\int_{-2}^7 12s^2+1\,ds$
10. $\int_0^1 (2x-1)^6\,dx$
11. $\int \frac{z}{(2-z)(2+z)}\,dz$
12. $\int 4te^{t^2}\,dt$
13. $\int \frac{6x}{3x^2+5}\,dx$
14. $\int_0^2 3y\sqrt{4-y^2}\,dy$
15. $\int \frac{x}{2x^2+3}\,dx$
16. $\int_0^1 6x^2e^{x^3}\,dx$
17. $\int \frac{6t}{2t^2+5}\,dt$
18. $\int \frac{e^{3x}}{4-e^{3x}}\,dx$
19. $\int_{-\ln(2)}^0 \frac{2e^x}{e^x+1}\,dx$
20. $\int \frac{3}{x\ln(x)}\,dx$ *
21. $\int_1^4 \frac{1}{2\sqrt{x}\,(1+\sqrt{x})^2}\,dx$ *
22. $\int_1^2 x(x-1)^4\,dx$ *
23. $\int \frac{2x}{(4x+5)^3}\,dx$ *

Answers

1. Ans. 0.5
2. $u=3x+5$, Ans. $\ln|3x+5|+C$
3. $u=4x+1)^8$, Ans. $\frac{1}{36}(4x+1)^9$
4. Ans. 108
5. $u=2x^2+1$, Ans. $\frac{1}{6}(2x^2+1)^6+C$
6. Ans. -1620
7. $u=2s^3+4s+3$, Ans. $\frac{\ln(3)}{2}$
8. $u=x^3+8$, Ans. $\ln|x^3+8|+C$
9. Ans. 1413
10. $u=2x-1$, Ans. $\frac{1}{7}$
11. $u=4-z^2$, Ans. $-\frac{1}{2}\ln|4-z^2|+C$
12. $u=t^2$, Ans. $2e^{t^2}+C$
13. $u=3x^2+5$, Ans. $\ln(3x^2+5)+C$
14. $u=4-y^2$, Ans. 8
15. $u=2x^2+3$, Ans. $\frac{1}{4}\ln(2x^2+3)+C$
16. $u=x^3$, Ans. $2e-2$
17. $u=2t^2+5$, Ans. $\frac{3}{2}\ln(2t^2+5)+C$
18. $u=4-e^{3x}$, Ans. $-\frac{1}{3}\ln|4-e^{3x}|+C$
19. $u=e^x+1$, Ans. $2\ln\left(\frac{4}{3}\right)$
20. $u=\ln(x)$, Ans. $3\ln|\ln|x||+C$
21. $u=1+\sqrt{x}$, Ans. $\frac{1}{6}$
22. $u=x-1$, Ans. $\frac{11}{30}$
23. $u=4x+5$, Ans. $-\frac{8x+5}{16(4x+5)^2}+C$
